

1 求极限 $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} =$

解: $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}x^2}{3x^2} = \frac{1}{6}$

2 求极限 $\lim_{x \rightarrow +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} =$

解: $\lim_{x \rightarrow +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \rightarrow +\infty} \frac{x^2}{1+x^2} = \lim_{x \rightarrow +\infty} \frac{2x}{2x} = 1$

3 求极限 $\lim_{x \rightarrow 0^+} \frac{\ln \tan 5x}{\ln \tan 3x}$

解: $\lim_{x \rightarrow 0^+} \frac{\ln \tan 5x}{\ln \tan 3x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{\tan 5x} \cdot \sec^2 5x \cdot 5}{\frac{1}{\tan 3x} \cdot \sec^2 3x \cdot 3} = 1$

4 求极限 $\lim_{x \rightarrow 0^+} \sin x \ln x =$

解: $\lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} \frac{\sin x}{\frac{1}{\ln x}} = \lim_{x \rightarrow 0^+} \frac{\cos x}{\frac{1}{(\ln x)^2} (-1) \cdot \frac{1}{x}} = \lim_{x \rightarrow 0^+} x \ln^2 x = 0$

$\lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} x \ln x$
 $= \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} (-x) = 0$

注, $\sin x \rightarrow 0$ 比 $\ln x \rightarrow \infty$ 要快, 故 $\lim_{x \rightarrow 0^+} \sin x \ln x = 0$

5 求极限 $\lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}}$

解: $\lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} e^{\frac{1}{x} \ln(e^x + x)}$

$= e^{\lim_{x \rightarrow 0} \frac{\ln(e^x + x)}{x}}$

$= e^{\lim_{x \rightarrow 0} \frac{\frac{e^x + 1}{e^x + x}}{1}} = e^{\lim_{x \rightarrow 0} \frac{e^x + 1}{e^x + x}} = e^{\frac{1+1}{1+0}}$

$= e^2$

4.2 P65

二 1 求极限 $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x \sin^2 x}$

$$\begin{aligned} \text{解: 原式} &= \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x^3} = \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{3x^2} = \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{6x} \\ &= \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{6} = \frac{1+1}{6} = \frac{1}{3} \end{aligned}$$

2 求极限 $\lim_{x \rightarrow 0} \frac{e^{x^2} - x^2 - 1}{x^2 \sinh^2 x}$

$$\begin{aligned} \text{解: } \lim_{x \rightarrow 0} \frac{e^{x^2} - x^2 - 1}{x^2 \sinh^2 x} &= \lim_{x \rightarrow 0} \frac{e^{x^2} - x^2 - 1}{x^4} = \lim_{x \rightarrow 0} \frac{2xe^{x^2} - 2x}{4x^3} = \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{2x^2} \\ &= \lim_{x \rightarrow 0} \frac{2xe^{x^2}}{4x} = \lim_{x \rightarrow 0} \frac{e^{x^2}}{2} = \frac{1}{2} \end{aligned}$$

3 求极限 $\lim_{x \rightarrow +\infty} \frac{\ln(a+be^x)}{\sqrt{a+bx^2}}$

(a, b > 0)

$$\begin{aligned} \text{解: 原式} &= \lim_{x \rightarrow +\infty} \frac{\ln(a+be^x)}{\sqrt{b}x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{a+be^x} \cdot be^x}{\sqrt{b}} \\ &= \lim_{x \rightarrow +\infty} \frac{be^x}{(a+be^x)\sqrt{b}} = \lim_{x \rightarrow +\infty} \frac{be^x}{\sqrt{b}be^x} = \frac{1}{\sqrt{b}} \end{aligned}$$

4 求极限 $\lim_{x \rightarrow 1} (1-x^2) \tan\left(\frac{\pi}{2}x\right)$

$$\text{解: 原式} = \lim_{x \rightarrow 1} \frac{1-x^2}{\cot\left(\frac{\pi}{2}x\right)} = \lim_{x \rightarrow 1} \frac{-2x}{(-\csc\frac{\pi}{2}) \cdot \frac{\pi}{2}} = \frac{4}{\pi}$$

5 求极限 $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

$$\begin{aligned} \text{解: 原式} &= \lim_{x \rightarrow 1} \frac{x \ln x - x + 1}{(x-1) \ln x} = \lim_{x \rightarrow 1} \frac{x \ln x - x + 1}{(x-1)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{\ln x + x \cdot \frac{1}{x} - 1}{2(x-1)} = \lim_{x \rightarrow 1} \frac{\ln x}{2(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x-1}{2(x-1)} = \frac{1}{2} \end{aligned}$$

注 $x \rightarrow 1$ 时, $\ln(x) = \ln(1+x-1) \sim x-1$

华东交通大学考试专用答题纸 (适用于匿名改卷)

课程: 4.2节(续) - 4.3节

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											
阅卷人											

考生注意: 答题时应注明题号, 并按题号顺序答题。

题号	答题内容	姓名
P67 4.2节		廖国勇
6	求极限 $\lim_{x \rightarrow 0} (\frac{1}{\sin^2 x} - \frac{1}{x^2})$ 解: 原式 = $\lim_{x \rightarrow 0} \frac{x^2 - \sin^2 x}{x^2 \sin^2 x} = \lim_{x \rightarrow 0} \frac{x^2 - \sin^2 x}{x^4} = \lim_{x \rightarrow 0} \frac{2x - \sin 2x}{4x^3}$ = $\lim_{x \rightarrow 0} \frac{2 - 2\cos 2x}{12x^2} = \lim_{x \rightarrow 0} \frac{2(1 - \cos 2x)}{12x^2}$ = $\lim_{x \rightarrow 0} \frac{2 \cdot \frac{1}{2}(2x)^2}{12x^2} = \frac{1}{3}$	
7	求极限 $\lim_{x \rightarrow +\infty} (x^2 + 2x)^{\frac{1}{x}}$ 解: 原式 = $\lim_{x \rightarrow +\infty} e^{\frac{1}{x} \ln(x^2 + 2x)} = e^{\lim_{x \rightarrow +\infty} \frac{\ln(x^2 + 2x)}{x}} = e^{\lim_{x \rightarrow +\infty} \frac{2x+2}{x^2+2x}}$ = $e^{\lim_{x \rightarrow +\infty} \frac{2}{2x+2}} = e^0 = 1$	
8	求极限 $\lim_{x \rightarrow +\infty} (\frac{2}{\pi} \arctan x)^x$ 解: 原式 = $\lim_{x \rightarrow +\infty} e^{x \ln(\frac{2}{\pi} \arctan x)} = e^{\lim_{x \rightarrow +\infty} \frac{\ln(\frac{2}{\pi} \arctan x)}{\frac{1}{x}}}$ = $e^{\lim_{x \rightarrow +\infty} \frac{\frac{1}{\frac{2}{\pi} \arctan x} \cdot \frac{2}{\pi} \cdot \frac{1}{1+x^2}}{-\frac{1}{x^2}}} = e^{\lim_{x \rightarrow +\infty} -\frac{2x^2}{\pi(1+x^2)}}$ = $e^{\lim_{x \rightarrow +\infty} -\frac{4x}{2\pi x}} = e^{-\frac{2}{\pi}}$	
9	求极限 $\lim_{x \rightarrow 0^+} (\cot x)^{\frac{1}{\ln x}}$ 解: 原式 = $e^{\lim_{x \rightarrow 0^+} \frac{\ln \cot x}{\ln x}} = e^{\lim_{x \rightarrow 0^+} \frac{\cot x \cdot (-\csc^2 x)}{x}} = e^{\lim_{x \rightarrow 0^+} x \frac{2-1}{\sin^2 x}}$ = e^{-1}	

题号

4.2节

P68 10 试确定常数 a, b 的值, 使函数 $f(x) = \begin{cases} b(1+\sin x) + a + 2, & x \geq 0 \\ e^{ax} - 1, & x < 0 \end{cases}$

处处可导

解: 由 $f'_+(0) = f'_-(0) \Rightarrow \lim_{\Delta x \rightarrow 0^+} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{f(0+\Delta x) - f(0)}{\Delta x}$

即 $\lim_{\Delta x \rightarrow 0^+} \frac{b(1+\sin \Delta x) + a + 2 - [b(1+\sin 0) + a + 2]}{\Delta x}$

$$= \lim_{\Delta x \rightarrow 0^-} \frac{e^{a\Delta x} - 1 - [b(1+\sin 0) + a + 2]}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x \rightarrow 0^+} \frac{b \sin \Delta x}{\Delta x} = b = \lim_{\Delta x \rightarrow 0^-} \frac{e^{a\Delta x} - a - b - 3}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x \rightarrow 0^-} (e^{a\Delta x} - a - b - 3) = 1 - a - b - 3 = 0 \Rightarrow a + b + 2 = 0$$

$$\Rightarrow \lim_{\Delta x \rightarrow 0^-} \frac{e^{a\Delta x} - a - b - 3}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{e^{a\Delta x} - 1}{\Delta x} = a = b$$

$$\Rightarrow \begin{cases} a + b + 2 = 0 \\ a = b \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = -1 \end{cases}$$

三 证明题, 设 $f''(x)$ 存在, 证明 $\lim_{h \rightarrow 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} = f''(x)$

$$\text{证明: } f(x+2h) = f(x) + f'(x) \cdot 2h + \frac{f''(x)}{2!} \cdot (2h)^2 + o(h^2)$$

$$= f(x) + f'(x) \cdot 2h + f''(x) \cdot 2h^2 + o(h^2)$$

$$f(x+h) = f(x) + f'(x) \cdot h + \frac{1}{2!} f''(x) h^2 + o(h^2)$$

$$\text{于是 } f(x+2h) - 2f(x+h) + f(x)$$

$$= f(x) + f'(x) \cdot 2h + f''(x) \cdot 2h^2 + o(h^2) - [2f(x) + 2f'(x)h + f''(x)h^2 + o(h^2)] + f(x)$$

$$= f''(x)h^2 + o(h^2)$$

$$\text{所以 } \lim_{h \rightarrow 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} = \lim_{h \rightarrow 0} \frac{f''(x)h^2 + o(h^2)}{h^2}$$

$$= f''(x)$$